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**Analysis Qualifying Exam Solutions**

ANALYSIS QUALIFYING EXAMINATION  
January 10, 2011

**SOLUTIONS**

1. Let  $A, B$  be arbitrary sets of real numbers such that  $a \leq 5$  for all  $a \in A, b \in B$ . Prove the following two statements are equivalent:  
(a)  $\sup A = \inf B$   
(b) For every  $\epsilon > 0$  there exist  $a \in A$  and  $b \in B$  such that  $b - a < \epsilon$ .

**Solution.** Assume first (a)  $\implies$  (b). Let  $\epsilon = \sup A - \inf B$ . Let  $a \in A$  be such that  $a > \sup A - \epsilon/2$ . Similarly, let  $b \in B$  be such that  $b < \inf B + \epsilon/2$ . Then  $b - a < \epsilon$ .

Conversely, assume the second condition. The fact that  $B$  is not empty and  $a \leq 5$  for all  $a \in A, b \in B$  implies  $A$  is bounded above and every element of  $B$  is an upper bound of  $A$ . Thus  $\sup A$  exists as a real number, and  $\sup A \leq b$  for all  $b \in B$ . Thus  $\sup A$  is a lower bound of  $B$ . It follows that  $\inf B \geq \sup A$ , and (a) holds. It remains to prove that  $\sup A = \inf B$ . Let  $\epsilon > 0$  be given. By our assumption, there exist  $a \in A, b \in B$  such that  $b - a < \epsilon$ . Then  $\inf B \leq b - a < \sup A + \epsilon$ . Since  $\sup A \leq \inf B$ , we have  $\sup A < \sup A + \epsilon$ . This implies  $\sup A < \inf B + \epsilon$ . Since  $\epsilon > 0$  was arbitrary, this implies  $\sup A \leq \inf B$ .

2. Let  $f : [0, \infty) \rightarrow \mathbb{R}$  be a functionally increasing and concave-down function. Prove that  $\lim_{x \rightarrow \infty} f(x) = 0$ .

**Solution.** This is an exercise from Page 1 of Real Mathematical Analysis. Let  $f' = \lim_{x \rightarrow \infty} f'(x)$ . Assume the conclusion to be false. Then there is a  $\delta > 0$  and a sequence  $\{x_n\}$  such that  $\lim_{n \rightarrow \infty} x_n = \infty$  and still  $f(x_n) \geq \delta$  for all  $n \in \mathbb{N}$ . Assume  $f'(x) > 0$  for all  $x \in \mathbb{R}$ . The case  $f'(x) \leq 0$  is similar. By uniform continuity there is  $\delta > 0$  such that  $|f(x) - f(y)| < \delta/2$  for  $|x - y| < \delta$ . It follows that  $f(x) > \delta/2$  for  $x \in (x_n - \delta, x_n + \delta)$  for all  $n \in \mathbb{N}$ . Hence  $\int_{x_n}^{x_n + \delta} f(x) dx \geq \delta/2 \delta$  for all  $n$ . Since  $\lim_{n \rightarrow \infty} x_n = \infty$ , by passing to a subsequence if necessary, we may assume that all the intervals  $(x_n, x_n + \delta)$  are mutually disjoint. Then since  $\lim_{x \rightarrow \infty} f(x) = 0$ ,  $\lim_{x \rightarrow \infty} \int_{x_n}^{x_n + \delta} f(x) dx = 0$ .

But  $\int_{x_n}^{x_n + \delta} f(x) dx \geq \delta/2 \delta$  for all  $n$ .

3. Let  $\phi : \mathbb{R} \rightarrow \mathbb{R}$  be defined by 
$$\phi(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ x^2 & \text{if } 0 < x < 1, \\ 1 & \text{if } x \geq 1. \end{cases}$$

Check the series  $\sum_{n=1}^{\infty} \frac{\phi(n)}{n^2}$ .