#Jenny

Finally I get this ebook, thanks for all these I can get now!

#Rio

Cool! I'am really happy

#Markus Jensen

I did not think that this would work, my best friend showed me this website, and it does! I get my most wanted eBook

#Hun Tsu

wtf this great ebook for free?!

#Che Salsa

My friends are so mad that they do not know how I have all the high quality ebook which they do not!

#Diego Butler

so many fake sites. this is the first one which worked! Many thanks

Problem 2.9

Show that the motion of an overcritically damped system due to initial displacement $u(0)$ and initial velocity $\dot{u}(0)$ is

$$u(t) = e^{-\zeta\omega_n t}\left(A_1 e^{-\omega_n\sqrt{\zeta^2-1}\,t} + A_2 e^{+\omega_n\sqrt{\zeta^2-1}\,t}\right)$$

where $\omega_n = \sqrt{\frac{k}{m}}$ and

$$A_1 = \frac{-u(0)\omega_n + \dot{u}(0) + \zeta\omega_n u(0)}{2\omega_n\sqrt{\zeta^2-1}}$$

$$A_2 = \frac{u(0)\omega_n + \dot{u}(0) + \zeta\omega_n u(0)}{2\omega_n\sqrt{\zeta^2-1}}$$

Solution:

Equation of motion:

$$m\ddot{u} + c\dot{u} + ku = 0 \qquad (a)$$

Dividing Eq. (a) through by m gives

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = 0 \qquad (b)$$

where $\zeta > 1$.

Assume a solution of the form $u(t) = e^{st}$. Substituting this solution into Eq. (b) yields

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)e^{st} = 0$$

Because e^{st} is never zero, the quantity within parentheses must be zero:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

or

$$s = \frac{-2\zeta\omega_n \mp \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2} = \left[-\zeta \mp \sqrt{\zeta^2-1}\right]\omega_n$$

The general solution has the following form:

$$u(t) = A_1 e^{\left[-\zeta+\sqrt{\zeta^2-1}\right]\omega_n t} + A_2 e^{\left[-\zeta-\sqrt{\zeta^2-1}\right]\omega_n t} \qquad (c)$$

where the constants A_1 and A_2 are to be determined from the initial conditions: $u(0)$ and $\dot{u}(0)$.

Evaluate Eq. (c) at $t = 0$:

$$u(0) = A_1 + A_2 = A_1 + A_2 = u(0) \qquad (d)$$

Differentiating Eq. (c) with respect to t gives

$$\dot{u}(t) = A_1\left[-\zeta+\sqrt{\zeta^2-1}\right]\omega_n e^{\left[-\zeta+\sqrt{\zeta^2-1}\right]\omega_n t} + A_2\left[-\zeta-\sqrt{\zeta^2-1}\right]\omega_n e^{\left[-\zeta-\sqrt{\zeta^2-1}\right]\omega_n t} \qquad (e)$$

Evaluate Eq. (e) at $t = 0$:

$$\dot{u}(0) = A_1\left[-\zeta+\sqrt{\zeta^2-1}\right]\omega_n + A_2\left[-\zeta-\sqrt{\zeta^2-1}\right]\omega_n$$

or

$$A_1\omega_n\left[-\zeta+\sqrt{\zeta^2-1}-\zeta-\sqrt{\zeta^2-1}\right] + \dot{u}(0) = \left[-2\sqrt{\zeta^2-1}\right]\omega_n u(0)$$

or

$$A_2 = \frac{\dot{u}(0) + \left[\zeta+\sqrt{\zeta^2-1}\right]\omega_n u(0)}{2\sqrt{\zeta^2-1}\,\omega_n} \qquad (f)$$

Substituting Eq. (f) in Eq. (d) gives

$$A_1 = u(0) - \frac{\dot{u}(0) + \left[\zeta+\sqrt{\zeta^2-1}\right]\omega_n u(0)}{2\sqrt{\zeta^2-1}\,\omega_n}$$

$$= \frac{2\sqrt{\zeta^2-1}\,\omega_n u(0) - \dot{u}(0) - \left[\zeta+\sqrt{\zeta^2-1}\right]\omega_n u(0)}{2\sqrt{\zeta^2-1}\,\omega_n}$$

$$= \frac{\zeta u(0) - \left[\zeta-\sqrt{\zeta^2-1}\right]u(0) - \dot{u}(0)}{2\sqrt{\zeta^2-1}\,\omega_n} \qquad (g)$$

© 2017 Pearson Education, Inc., Hoboken, NJ. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

[Download PDF version of :]

Dynamics Of Structures Chopra Solution